

Detecting Causal Interactions between Atlantic and Pacific Ocean Variability

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Sapienza Università di Roma

Climate Change

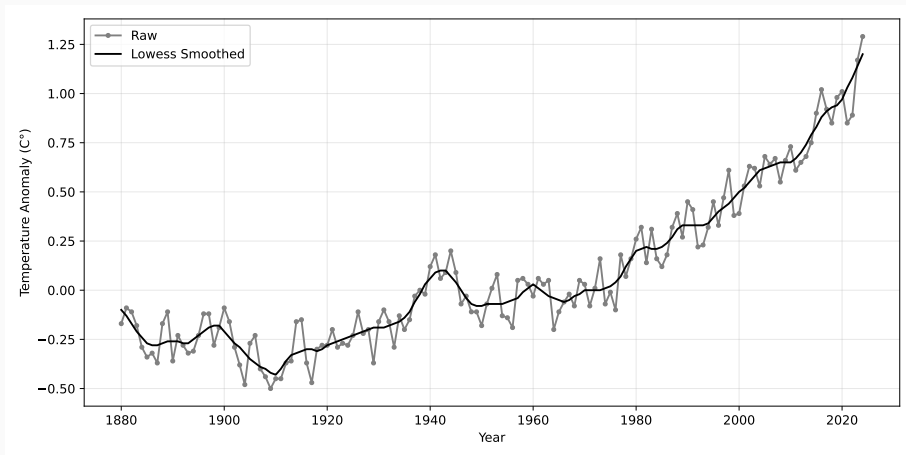
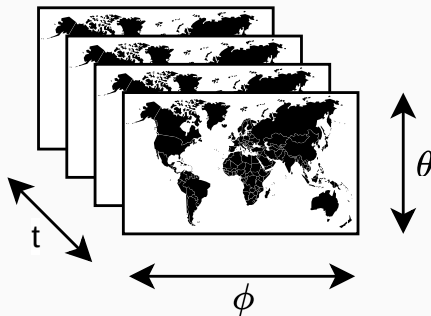


Figure 1: Change in global surface temperature compared to the long-term average from 1951 to 1980 [9].

Sea Surface Temperature (SST)



Name	Resolution t	Resolution ϕ, θ	Time Range	Source
HadISST	Monthly	$\phi = [-180, 180], \theta = [-90, 90]$	1871 – present	[12]
ERSSTV5	Monthly	$\phi = [-180, 180], \theta = [-90, 90]$	1854 – present	[5]

Table 1: Summary of the HadISST and ERSSTV5 datasets.

Climate Indices

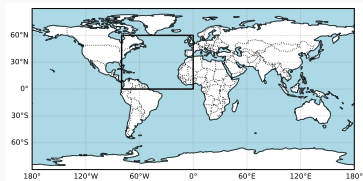


Figure 2: AMV Boundaries

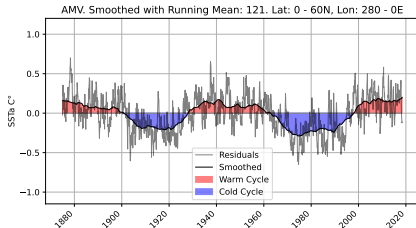


Figure 3: Atlantic Multidecadal Variability (AMV)

Climate Indices

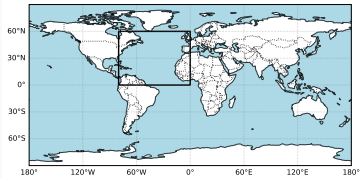


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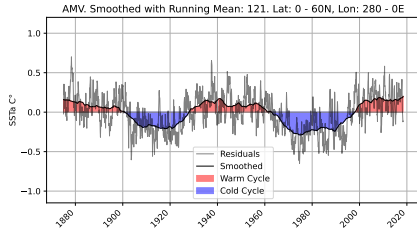


Figure 3: Atlantic Multidecadal Variability (AMV)

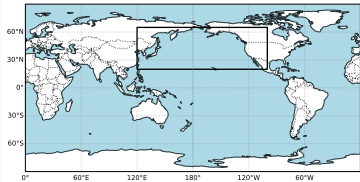


Figure 4: PDO Boundaries

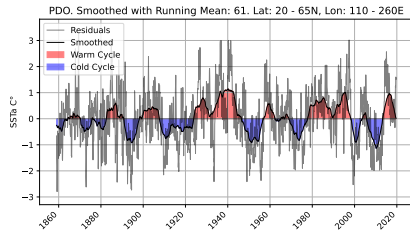


Figure 5: Pacific Decadal Oscillation (PDO)

What is the relationship between the AMV and PDO indices?

Pacific Modes of Variability

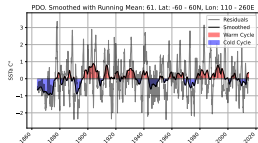


Figure 6: ENSO

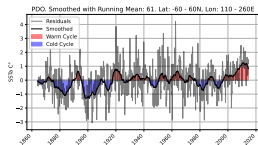


Figure 7: SEPM

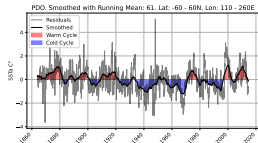


Figure 8: SPDOL

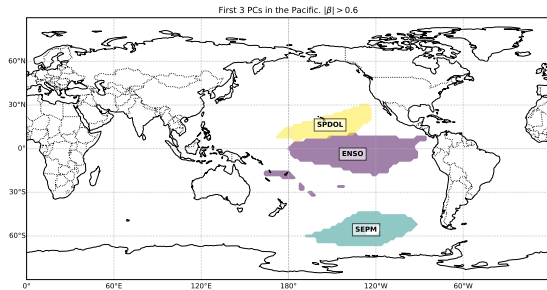
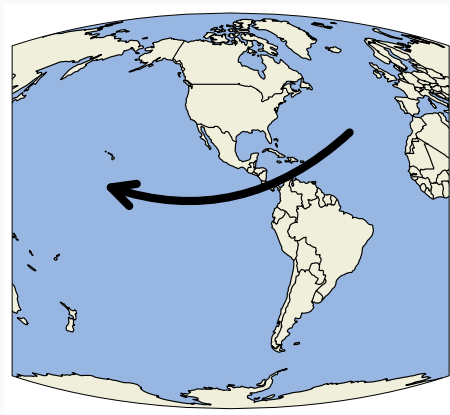


Figure 9: The first 3 varimax rotated principal components in the Pacific 60S to 60N, 110E to 100W.

Existing literature

Previous research show a connection from the AMV to the Pacific [16, 10, 2, 15, 1, 3], but use **model simulations**.

We want to use **observational data**.



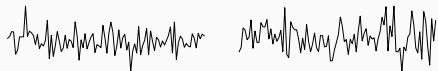
Causal Discovery Networks

Learn a **causal graph** from **observational** data. In the causal graph, nodes are observations, and edges are causal relationships.



(a) A

(b) B

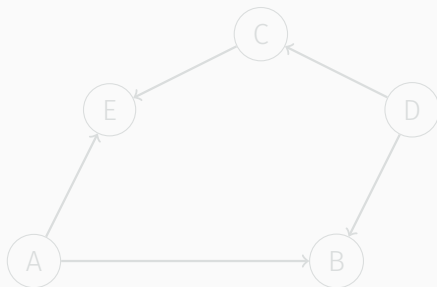


(c) C

(d) D



(e) E



$$A = f_A()$$

$$B = f_B(A, D)$$

$$C = f_C(D)$$

$$D = f_D()$$

$$E = f_E(A, C)$$

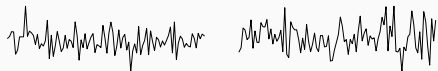
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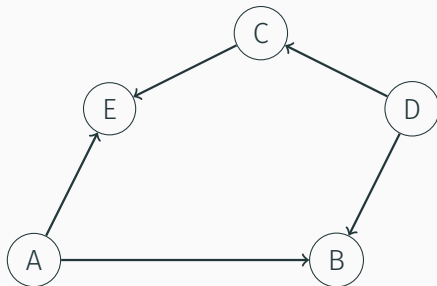


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Granger Causality

Check if f_{full} has lower prediction error than f_{red} :

$$f_{\text{red}} = a_0 + \sum_{i=1}^p a_i Y_{t-i} + \epsilon_t, f_{\text{full}} = b_0 + \sum_{i=1}^p b_i Y_{t-i} + \sum_{j=1}^q c_j X_{t-j} + \eta_t \quad (1)$$

PCMCi+

Use conditional independence (CI) tests to iteratively remove links from a complete graph.

$$X \perp\!\!\!\perp Y \mid Z \quad (2)$$

LKIF

Use a maximum likelihood estimator (MLE) to estimate the rate of entropy transfer between variables in the system. A link exists if the information transfer is statistically significant at level α .

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Apply Granger Causality, PCMCi+, LKIF to two networks, at $\alpha = 0.05$, $\tau = 12$:

Network 1

- AMV
- PDO

Network 2

- AMV
- PDO
- PC1 Varimax Rotated (ENSO)
- PC2 Varimax Rotated (SEPM)
- PC3 Varimax Rotated (SPDOL)

Result Graphs

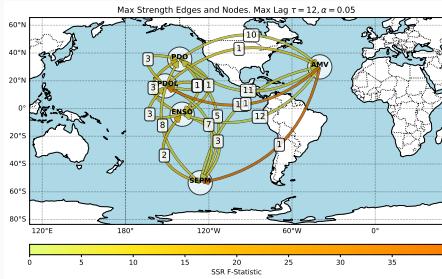


Figure 11: Granger Causality

Result Graphs

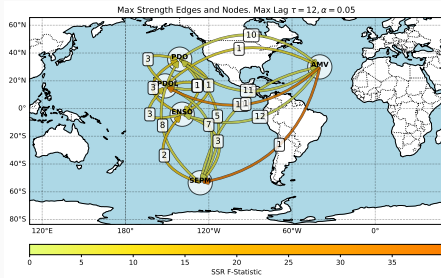


Figure 11: Granger Causality

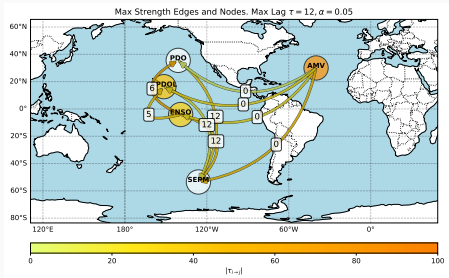


Figure 12: LKIF

Result Graphs

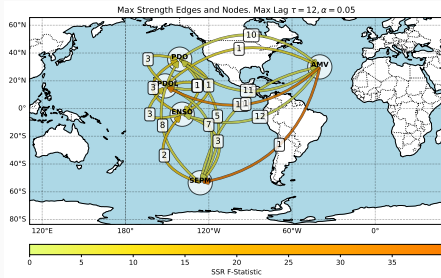


Figure 11: Granger Causality

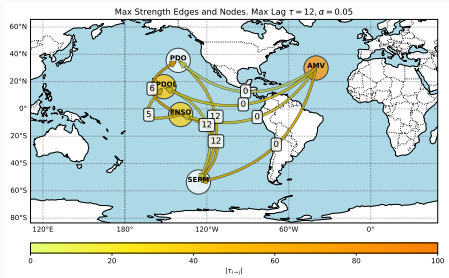


Figure 12: LKIF

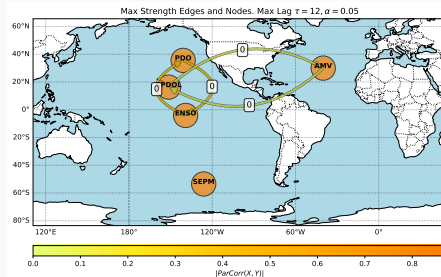


Figure 13: PCMCi+

Result Edge Strengths - from AMV

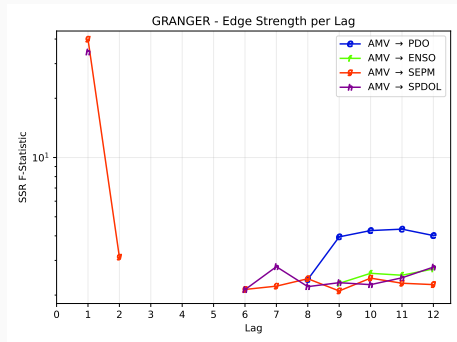


Figure 14: Granger Causality

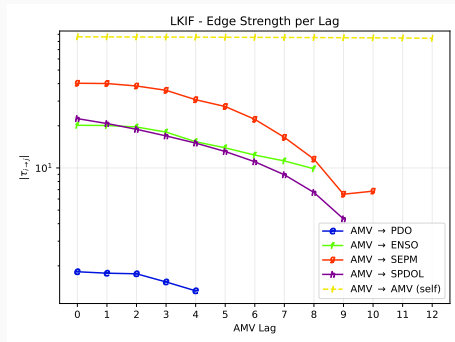


Figure 15: LKIF

Result Edge Strengths - to PDO

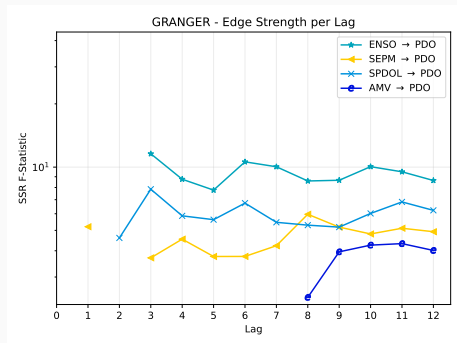


Figure 16: Granger Causality

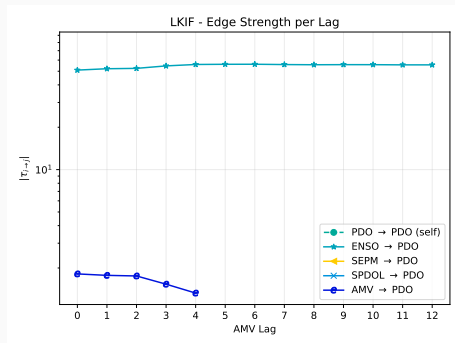


Figure 17: LKIF

- Granger Causality and LKIF find links from AMV to PDO → changes in the Atlantic are followed by changes in the Pacific.
- Links from AMV to PDO tend to be weaker than links from other Pacific modes to PDO (Figure 16, Figure 17) → PDO is be modulated by AMV via other Pacific components.

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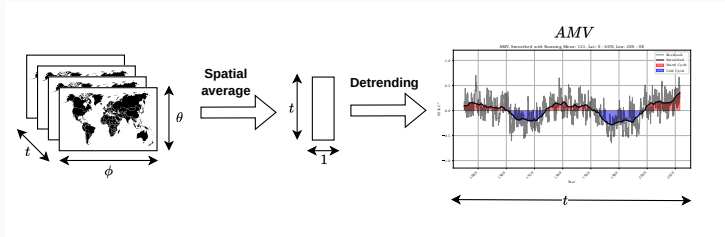


Figure 18: Atlantic Multidecadal Variability (AMV)

Climate Indices

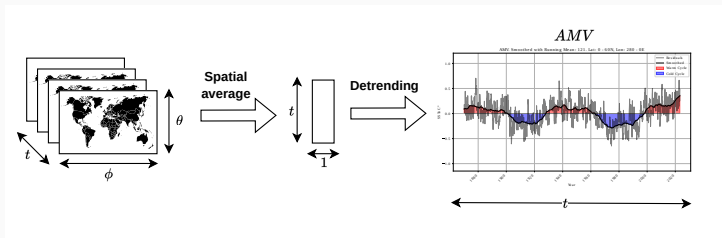


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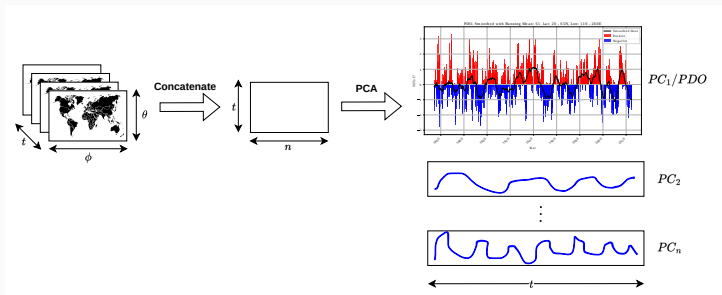
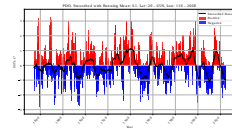
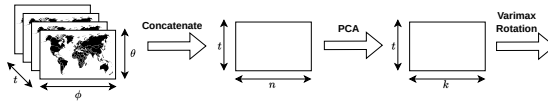


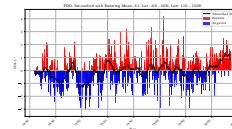
Figure 19: Pacific Decadal Oscillation (PDO)

Varimax Rotation

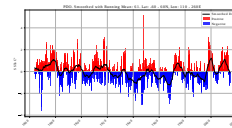
Enrich the system by adding more Pacific variables, computed via PCA followed by a Varimax rotation.



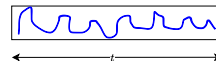
PC_1



PC_2



PC_3



PC_k

Granger Causality [4, 11]

Given timeseries X (cause) and Y (effect), fit **two** regression models to predict Y :

$$f_{red} = a_0 + \sum_{i=1}^p a_i Y_{t-i} + \epsilon_t \quad (3)$$

$$f_{full} = b_0 + \sum_{i=1}^p b_i Y_{t-i} + \sum_{j=1}^q c_j X_{t-j} + \eta_t \quad (4)$$

Compare using F-test where F is:

$$F = \frac{SSE_{red} - SSE_{full} / (r - s)}{SSE_{full} / (T - r)} \quad (5)$$

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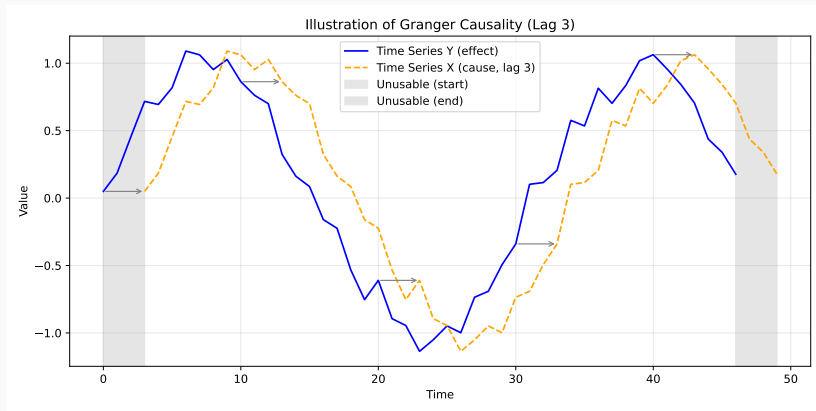


Figure 21

Use **Conditional Independence** (CI) tests to determine if two variables are independent **given** a set Z .

$$X_i \perp\!\!\!\perp X_j | Z \quad (6)$$

1. Start with fully connected graph.
2. Discover lagged parents $\widehat{B}_t^-(X_t^i)$.
3. Condition on lagged parents $\widehat{B}_t^-(X_t^i)$ and contemporaneous connections S to refine parent set.

$$MCI : X_i^{t-\tau} \perp\!\!\!\perp X_j^t \mid S, \widehat{B}_-(X_j^t) \setminus \{X_i^{t-\tau}\}, \widehat{B}_-^{t-\tau}(X_i^{t-\tau}) \quad (7)$$

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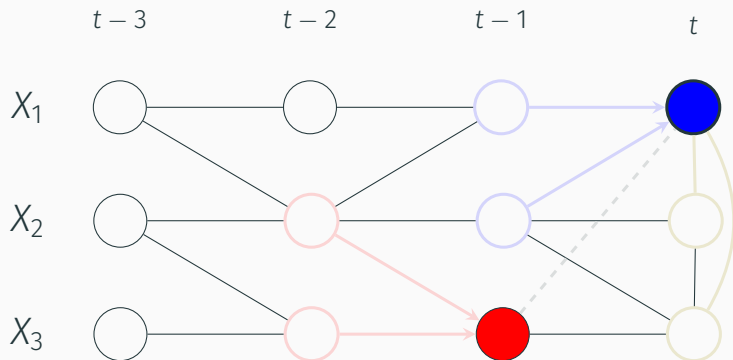
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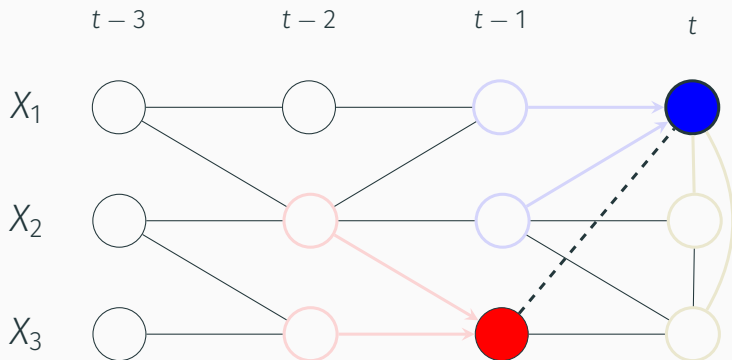
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PCMCI+ Timeseries Graph



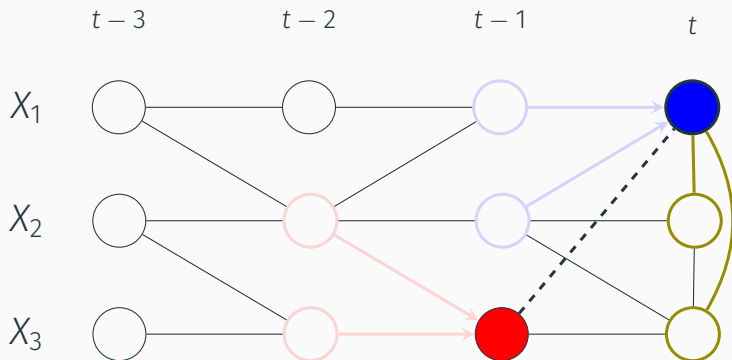
$$MCI : \mathbf{x}_3^{t-1} \perp\!\!\!\perp \mathbf{x}_1^t \mid \mathbf{s}, \hat{B}_{-}^{t-1}(\mathbf{x}_3^{t-1}), \hat{B}_{-}^t(\mathbf{x}_1^t) \setminus \{\mathbf{x}_3^{t-1}\} \quad (8)$$

PCMCI+ Timeseries Graph



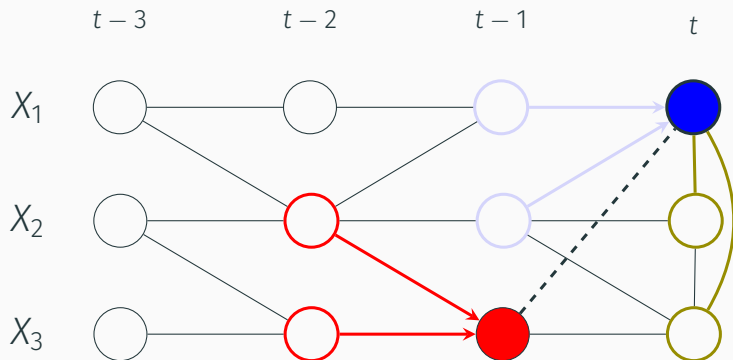
$$MCI : \mathbf{x}_3^{t-1} \perp\!\!\!\perp \mathbf{x}_1^t \mid \mathbf{s}, \hat{B}_{-}^{t-1}(\mathbf{x}_3^{t-1}), \hat{B}_{-}^t(\mathbf{x}_1^t) \setminus \{\mathbf{x}_3^{t-1}\} \quad (8)$$

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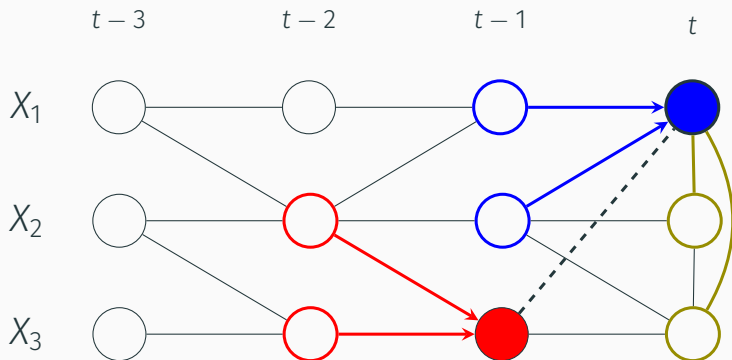
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Measure the **information flow** between variables, using a Maximum Likelihood Estimator (MLE).

$$\hat{\tau}_{i \rightarrow j} = \frac{1}{\det \mathbf{C}} \sum_{k=1}^d \Delta_{jk} C_{k,di} \frac{C_{ij}}{C_{ii}} \quad (9)$$

Normalised with

$$Z_i = \sum_{k=1}^d \left| \hat{\tau}_{k \rightarrow i} \right| + \left| \frac{dH_i^{\text{noise}}}{dt} \right| \quad (10)$$